遅延フィードバックによって制御された区分線形離散力学系における過渡カオスと間欠カオス

潮　　俊光†

† 大阪大学大学院基礎工学研究科　〒 560–8531 豊中市待兼山町 1–3
E-mail: †ushio@sys.es.osaka-u.ac.jp

あらまし　遅延フィードバック制御は，過去の状態との差を基にしたフィードバック制御法で，安定化したい周期軌道を精確に求めなくても安定化できる．遅延フィードバック制御には，奇数制約と呼ばれる安定化できるための制約が知られており，それを克服するための方法については多くの研究がある，しかし，制御したときの大域的な性質については，最近，山末と引原によってダフィング方程式において研究がある程度である．本報告では，1 次元区分的離散力学系を対象にその大域的性質を調べる．さらに，遅延フィードバックによって過渡カオスや間欠カオスを発生させれることを明らかにする．

キーワード　遅延フィードバック制御，離散力学系，カオス制御，過渡カオス，間欠カオス

Transient Chaos and Intermittency in Delayed Feedback Controlled Piecewise-Linear Discrete-Time Systems

Toshimitsu USHIO†

† Graduate School of Engineering Science, Osaka University　Machikaneyama 1–3, Toyonaka-shi, 560–8531 Japan
E-mail: †ushio@sys.es.osaka-u.ac.jp

Abstract　Delayed feedback control is based on the difference between a past state and the current one, and can stabilize an unstable periodic orbit without its precise calculation. It is known that the delayed feednack control has a limitaiton called the odd number limitation for stabilization, and many studies have been done to overcome the limitation. However, its global properties of controlled systems have not been paid much attention to. Recenlty, Yamasue and Hikihara analized a domain of attraction in a delayed feedback controlled Duffing equation. In this report, we investigate global properties of a one-dimensional piece-wise linear discrete-time system. Moreover, we show thatboth tranisient chaos and intermittent chaos are obsereved by delayed feedback control.

Key words　delayed feedback control, discrete-time system, controlling chaos, transient chaos, intermittent chaos

We consider the following one-dimensional piecewise-linear systems:

$$x(k+1) = f(x(k)), \tag{1}$$

where

$$f(x) = \begin{cases} -\beta x - \beta(1+\epsilon) - 1 + \epsilon\alpha & \text{if } x < -1-\epsilon, \\ -\alpha x - \alpha - 1 & \text{if } -1-\epsilon \leqq x < -\delta, \\ \frac{\alpha(1-\delta)+1}{\delta} x & \text{if } -\delta \leqq x < \delta, \\ -\alpha x + \alpha + 1 & \text{if } \delta \leqq x < 1+\epsilon, \\ -\beta x + \beta(1+\epsilon) + 1 - \epsilon\alpha & \text{if } 1+\epsilon \leqq x, \end{cases} \tag{2}$$

and α, β, δ, and ϵ are positive parameters with $0 < \delta < 1$. MOreover, ϵ is small enough to satisfy $1+\epsilon < \alpha(1-\delta)+1$. Equation (1) has three fixed points -1, 0, and 1, which are independent of the parameters. It is easily shown that there are two invariant intervals if $f^2(\delta) < 0$, that is,

$$\alpha\beta(\delta-1) + \epsilon(\beta-\alpha) + 1 < 0, \tag{3}$$

and a crisis occurs when $f^2(\delta) = 0$ holds. After the occurence of the crisis, one invariant interval including all the fixed points exists.

We control Eq. (1) using delayed feedback. The controlled system is given by

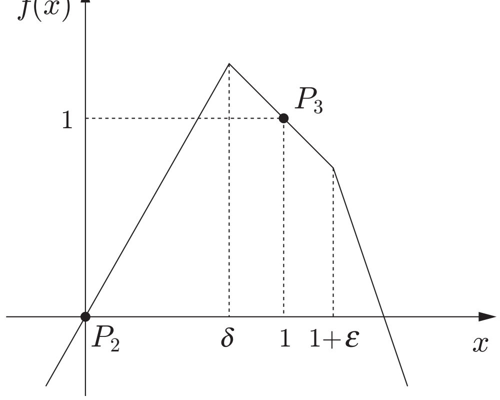

図 1 one-dimensional map.

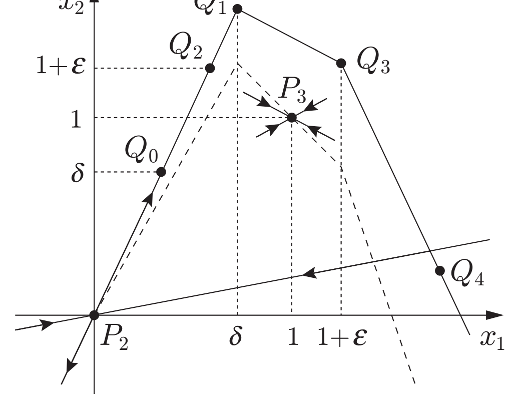

図 2 Stable and unstable manifolds in controlled system.

$$x(k+1) = f(x(k)) + K(x(k-1) - x(k)). \tag{4}$$

The above equation can be written as follows:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ K & -K \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ f(x_2(k)) \end{bmatrix}, \tag{5}$$

where $x_1(k) = x(k-1)$ and $x_2(k) = x(k)$. In the following, a function $F_K : \Re^2 \to \Re^2$ with the control parameter K is defined as follows:

$$F_K(X) = \begin{bmatrix} 0 & 1 \\ K & -K \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ f(x_2) \end{bmatrix},$$

where $X = [x_1,\ x_2]^T$. For simplicity, if a point A is mapped to a point B by F_K, we will write $B = F_K(A)$. Equation (5) has three fixed point, $P_1 = [-1\ \ -1]^T$, $P_2 = [0\ 0]^T$, and $P_3 = [1\ 1]^T$. Its linearized system around the fixed point $P_i (i = 1, 2, 3)$ is given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ K & a_i - K \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \tag{6}$$

where $a_1 = a_3 = -\alpha$ and $a_2 = \frac{\alpha(1-\delta)+1}{\delta}$. The characteristic equation of the linearized system around P_i is given by

$$\mu^2 - (a_i - K)\mu - K = 0 \tag{7}$$

Thus, the fixed points P_i are aymptotically stable if the following equation holds.

$$\min(1,\ \frac{a_i + 1}{2}) > K > -1,\ a_i < 1. \tag{8}$$

The fixed point P_2 is always unstable since $\alpha > 0$. It is noted that this fact is also driven from the odd number limitation. Equation (7) has two real roots $\mu_{i,1}$ and $\mu_{i,2}$ ($|\mu_{i,1}| < |\mu_{i,2}|$) if K is close to 0. Obviously, if $K = 0$, then $\mu_{i,1} = 0$ and $\mu_{i,2} = a_i$. Moreover, the local stable and the local unstable manifold of the fixed point P_2 are on the lines $x_2 = 0$ and $x_2 = a_2 x_1$, respectively, and the orbit starting from any points satisfies $x_2 = F(x_1)$ after one step. In other words, the unstable manifold of the fixed point P_2 satisfies $x_2 = F(x_1)$, and the other fixed points P_1 and P_3 belongs to the unstable manifold. Thus, if $\alpha > 1$, then both P_1 and P_3 are saddle points and their unstable manifolds are coincident with that of P_2.

Now, we consider the case that $\alpha > 1$. Then, to stabilize the fixed points P_1 and P_2, the control gain should be negative. So we suppose that K is negative and close to zero. Then the unstable manifolds are perturbed by the control input. The local unstable manifold of P_2 forms a segment given by

$$x_2 = \mu_{2,2} x_1, \tag{9}$$

where $|x_1| < \delta$, and its local stable manifold forms a segment given by

$$x_2 = \mu_{2,1} x_1, \tag{10}$$

where $|x_1| < \delta$. We assume that ϵ is small enough to satisfy that $\mu_{2,2}\delta > 1 + \epsilon$. Then, the local unstable manifold is extended as shown in Fig. 2. We will define the following poits:

$$Q_0 = \begin{bmatrix} \mu_{2,2}^{-1}\delta \\ \delta \end{bmatrix}, \tag{11}$$

$$Q_1 = \begin{bmatrix} \delta \\ \mu_{2,2}\delta \end{bmatrix}, \tag{12}$$

$$Q_2 = \begin{bmatrix} \mu_{2,2}^{-1}(1+\epsilon) \\ 1+\epsilon \end{bmatrix}, \tag{13}$$

$$Q_3 = \begin{bmatrix} 1+\epsilon \\ (1+\epsilon)K(\mu_{2,2}^{-1} - 1) - \epsilon\alpha + 1 \end{bmatrix}, \tag{14}$$

$$Q_4 = \begin{bmatrix} \mu_{2,2}\delta \\ K\delta(1-\mu_{2,2}) + \beta(1+\epsilon-\mu_{2,2}\delta) + 1 - \epsilon\alpha \end{bmatrix}. \tag{15}$$

Then, we have

$$Q_1 = F_K(Q_0),$$
$$Q_4 = F_K(Q_1),$$
$$Q_3 = F_K(Q_2),$$

and the unstable manifold consists of the segments P_2Q_1, Q_1Q_3, and Q_3Q_4.

We will consider an effect of the feedbac gain K on the slope $\mu_{2,2}$ of the segment P_2Q_1. Let $\mu_{2,2} = a_2 + \Delta\mu_{2,2}$, where $\Delta\mu_{2,2}$ is a pertubation by the control gain K. Then, by Eq. (7), we have

$$\mu_{2,2} = \frac{K}{\Delta\mu_{2,2} + K}. \tag{16}$$

The Taylor expansion of $\mu_{2,2}$ around $K = 0$ is as follows:

$$\mu_{2,2} = \frac{a_2 - K + \sqrt{(a_2 - K)^2 + 4K}}{2} \tag{17}$$

$$= a_2 + \frac{1 - a_2}{a_2} K + \textit{higher terms.} \tag{18}$$

Recall that K is negative to stabilize the fixed point P_1 and P_3. Thus, if P_2 is unstable, that is, $a_2 > 1$, then $\Delta\mu_{2,2} > 0$ when the stabilization is achieved. This means that P_2 is more unstable when P_1 and P_3 are stabilized by the delayed feedback. Moreover, the slope of unstable manifold of P_2 in the controlled system is steeper than that in the uncontrolled system.

Next, the segment Q_1Q_3 is given by

$$x_2 = \frac{(1+\epsilon)K(\mu_{2,2}^{-1} - 1) - \epsilon\alpha + 1 - \mu_{2,2}\delta}{1 + \epsilon - \delta}(x_1 - \delta).$$

Using Eq. (16), the above equation is

$$x_2 = (\Delta\mu_{2,2} - \alpha)x_1 + \alpha + 1. \tag{19}$$

Thus, the slope of the segment Q_1Q_3 in the controlled system is smaller than that in uncontrolled one.

From these discussions, the unstable manifold of P_2 is shifted upward in the first quadrant. Similarly, it is shifted downward in the third quadrant. Thus, shown in Fig. 2 is an illustration of the pertubation of the unstable manifold when P_3 is stabilized by the delayed feedback. From Fig. 2, when the segment Q_3Q_4 intersects the stable manifold of P_2, a tranverse homoclinic point exists and chaos can be observed. It is easily that the intersection exists if the following condition holds:

$$\alpha\beta(\delta - 1) + \epsilon(\beta - \alpha) + 1 \leqq \mu_{2,1}\mu_{2,2}\delta - K\delta(1 - \mu) + \alpha\beta\Delta\mu_{2,2} \tag{20}$$

The above equation is equal to Eq. (3) when $K = 0$. Moreover, when the equality holds, a crisis occurs.

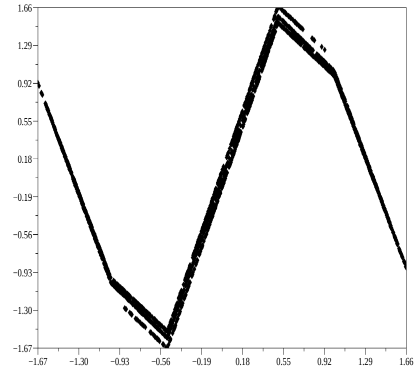

図 3 Phase portrait when chaotic transient occurs($K = -0.06,\ \alpha = 1.1,\ \beta = 3,\ \epsilon = 0.01,\ \delta = 0.5$)

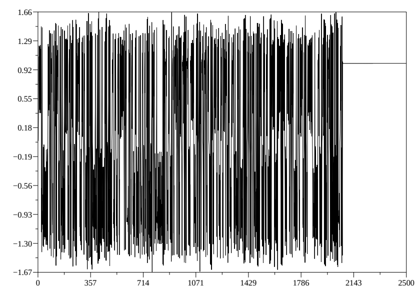

図 4 Chaotic transient($K = -0.06,\ \alpha = 1.1,\ \beta = 3,\ \epsilon = 0.01,\ \delta = 0.5$)

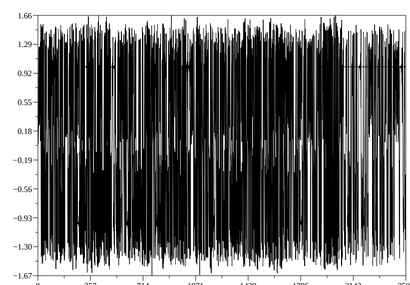

図 5 Behavior of uncontrolled system($K = -0,\ \alpha = 1.1,\ \beta = 3,\ \epsilon = 0.01,\ \delta = 0.5$)

文　献

[1] 山末 耕平, 引原 隆士, 時間遅れフィードバック制御された磁気弾性系の解の収束と 不変多様体, 電子情報通信学会 非線形問題研究会, NLP2003-32, 公立はこだて未来大 学, July 14, 2003.

[2] K. Yamasue and T. Hikihara, Domain of attraction for stabilized orbits in time delayed feedback controlled Duffing systems, Phys. Rev. E 69, 056209 (2004).

[3] E. Ott, C. Grebogi, and J. A. Yorke, "Controlling chaos," *Phys. Rev. Lett.*, vol. 64, no. 11, pp. 1196–1199, 1990.

[4] F. J. Romeiras, C. Grebogi, E. Ott, and W. P. Dayawansa, "Controlling chaotic dynamical systems," *Physica*, vol. D58 pp. 165–192, 1992.

[5] T. Shinbrot, C. Grebogi, E. Ott and J. A. Yorke, "Using small perturbations to control chaos," *Nature*, vol. 363 pp. 411–417, 1993.

[6] G. Chen and X. Dong, "On feedback control of chaotic continuous-time systems," *IEEE Trans. Circuits and Systems-I*, vol. 40, no. 9, pp. 591–601, 1993.

[7] G. Chen and X. Dong, "From chaos to order – perspectives and methodologies in controlling chaotic nonlinear dynamical systems," *Int. J. Bif. and Chaos*, vol. 3, no. 6, pp. 1363–

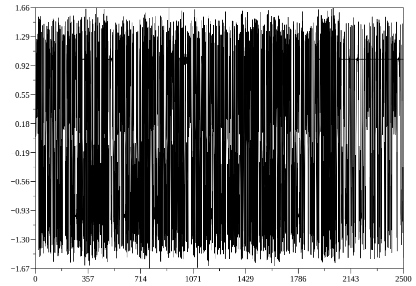

図 6 Phase portrait of uncontrolled system($K = -0,\ \alpha = 1.1,\ \beta = 3,\ \epsilon = 0.01,\ \delta = 0.5$)

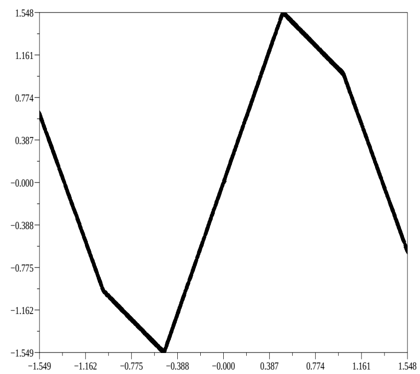

図 7 Phase portrait of uncontrolled system($K = -0,\ \alpha = 1.1,\ \beta = 3,\ \epsilon = 0.01,\ \delta = 0.5$)

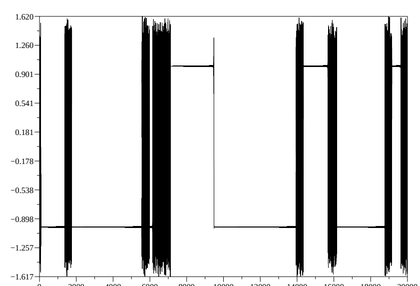

図 8 Intermittent chaos ($K = -0.04,\ \alpha = 1.0806,\ \beta = 3,\ \epsilon = 0.01,\ \delta = 0.5$)

1409, 1993.

[8] K. Pyragas, "Continuous control of chaos by self-controlling feedback," *Phys. Lett. A*, vol. 170, pp. 421–428, 1992.

[9] K. Pyragas and A. Tamaševičius, "Experimental control of chaos by delayed self-controlling feedback," *Phys. Lett. A*, vol. 180, pp. 99-102, 1993.

[10] Z. Qu, G. Hu and B. Ma, "Controlling chaos via continuous feedback," *Phys. Lett. A*, vol. 178, pp. 265–270, 1993.

[11] T. Kawagoshi and T. Hikihara, "An experimental study on stabilization of periodic motion in magneto-elastic chaos," *Tech. Report of IEICE, Japan*, NLP94-80, 1994.

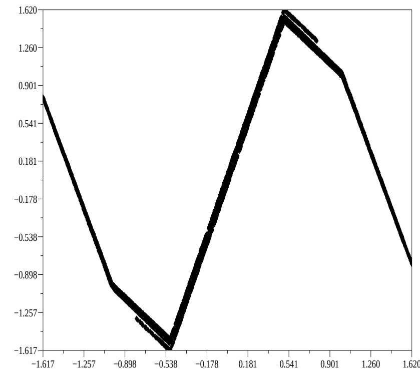

図 9 Phase portrait when intermittent chaos occurs($K = -0.04,\ \alpha = 1.0806,\ \beta = 3,\ \epsilon = 0.01,\ \delta = 0.5$)

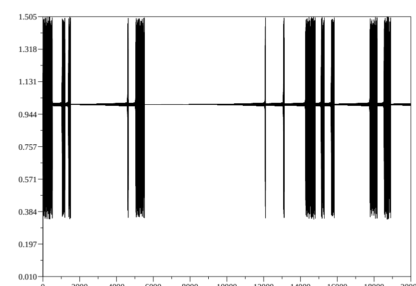

図 10 Intermittent chaos where one quasi-steady state exists($K = -0.04,\ \alpha = 1.0806,\ \beta = 1.395,\ \epsilon = 0.01,\ \delta = 0.5$)