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動的なデフォルト制御を用いた分散スーパバイザ制御

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あらまし　従来提案されている離散事象システムの分散スーパバイザ制御構造では，可制御事象に対するデフォルト制御動作は静的とされていた．本研究では，動的なデフォルト制御を用いた分散スーパバイザ制御構造を提案し，その制御構造のもとで分散スーパバイザが存在するための必要十分条件を導出する．そして，従来の制御構造ではスーパバイザが構成できないが，提案した制御構造においてスーパバイザが構成できるような言語が存在することを示す．
キーワード　離散事象システム，分散スーパバイザ制御，動的デフォルト制御，動的共可観測性

Decentralized Supervisory Control Using Dynamic Default Control

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Abstract　The conventional decentralized supervisory control architectures for discrete event systems assume that default control of controllable events is static. In this paper, we propose a new decentralized supervisory control architecture using dynamic default control of controllable events. We present necessary and sufficient conditions for the existence of a decentralized supervisor in the proposed architecture. Then, we give an example of a language that is achieved in the proposed architecture, but not in the conventional architectures using static default control.
Key words　discrete event system, decentralized supervisory control, dynamic default control, dynamic co-observability

1. Introduction

In this paper, we study decentralized supervisory control of discrete event systems (DESs). In the original decentralized control architecture studied in [1], [2], default control of controllable events is "enable", and a decentralized supervisor issues the disable decision in order to disable an illegal controllable event. Yoo and Lafortune have proposed a decentralized control architecture where, for each controllable event, default control is fixed as "enable" or "disable", and a decentralized supervisor issues the enable and disable decisions [3]. Recently, they have extended this architecture by introducing conditional decisions [4].

In the control architectures mentioned above, default control of controllable events is *static*, i.e., for each controllable event, default control is fixed under operation. In this paper, we propose a new decentralized supervisory control architecture using *dynamic* default control of controllable events. We present necessary and sufficient conditions for the existence of a decentralized supervisor in the proposed architecture. Then, we give an example of a language that is achieved in the proposed architecture, but not in the conventional architectures using static default control.

2. Preliminaries

We consider a DES modeled by an automaton $G = (Q, \Sigma, \delta, q_0, Q_m)$, where Q is the set of states, Σ is the finite set of events, a partial function $\delta : \Sigma \times Q \to Q$ is the transition function, $q_0 \in Q$ is the initial state, and $Q_m \subseteq Q$ is the set of marked states. Let Σ^* be the set of all finite strings of elements of Σ, including the empty string ε. The function δ can be generalized to $\delta : \Sigma^* \times Q \to Q$ in the natural way. The generated and marked languages of G are denoted by $L(G)$ and $L_m(G)$, respectively. Let $K \subseteq \Sigma^*$ be a language. We denote the set of all prefixes of strings in K by $\overline{K}$.

For supervisory control [5], the event set Σ is partitioned into two disjoint subsets Σ_c and Σ_{uc} of controllable and uncontrollable events, respectively. A supervisor is defined by a map $S : L(G) \to 2^\Sigma$ such that $\Sigma_{uc} \subseteq S(s)$ for any $s \in L(G)$. The generated language $L(S/G)$ under the control action of S is defined inductively as follows:

- $\varepsilon \in L(S/G)$,
- $(\forall s \in L(S/G), \forall \sigma \in \Sigma)$
  $s\sigma \in L(S/G) \Leftrightarrow [s\sigma \in L(G)] \wedge [\sigma \in S(s)]$.

The marked language $L_m(S/G)$ under the control action of S is defined as $L_m(S/G) := L(S/G) \cap L_m(G)$. If $L(S/G) = \overline{L_m(S/G)}$, then S is said to be *nonblocking*.

A language $K \subseteq L(G)$ is said to be

- $L_m(G)$-closed [5] if $\overline{K} \cap L_m(G) = K$.
- controllable (with respect to Σ_{uc}) [5] if $\overline{K}\Sigma_{uc} \cap L(G) \subseteq \overline{K}$.

We assume that a control specification is described by a nonempty sublanguage $K \subseteq L_m(G)$. There exists a nonblocking supervisor $S : L(G) \to 2^\Sigma$ such that $L_m(S/G) = K$ if and only if K is $L_m(G)$-closed and controllable [5].

In this paper, we consider decentralized supervisory control where n local supervisors control the system so that the controlled behavior satisfies a (global) specification. Let Σ_{ic} be the set of locally controllable events for an ith local supervisor S_i $(i \in I := \{1, 2, \cdots, n\})$ such that

$$\Sigma_c = \bigcup_{i \in I} \Sigma_{ic}.$$

For each controllable event $\sigma \in \Sigma_c$, we define the index set $In(\sigma)$ by $In(\sigma) = \{i \in I | \ \sigma \in \Sigma_{ic}\}$. We assume that an ith local supervisor S_i $(i \in I)$ observes an event through the local observation mask $M_i : \Sigma \to \Delta_i \cup \{\varepsilon\}$ where Δ_i is the locally observed event set [1].

In the Enabling/Disabling architecture for decentralized supervisory control, the controllable event set Σ_c is partitioned into two disjoint subsets $\Sigma_{c,e}$ and $\Sigma_{c,d}$ [3]. A controllable event in $\Sigma_{c,e}$ (respectively, $\Sigma_{c,d}$) is enabled (respectively, disabled) by default, and disabled (respectively, enabled) if a supervisor decides to disable (respectively, enable) it. The default control is *static* in the sense that the partition $\{\Sigma_{c,e}, \Sigma_{c,d}\}$ of Σ_c is fixed under the operation.

Each local supervisor S_i^a with the two decisions "enable" and "disable" is defined by a map $S_i^a : M_i(L(G)) \to 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}}$, where

$$S_i^a(M_i(s)) = (e_i(M_i(s)), d_i(M_i(s)))$$

for $s \in L(G)$. The components of $S_i^a(M_i(s))$ represent the following control decisions on Σ_{ci}:

$$e_i(M_i(s)) \ : \text{events that are enabled,}$$
$$d_i(M_i(s)) \ : \text{events that are disabled.}$$

By using these two kinds of local decisions, the decentralized supervisor is defined by a map $\{S_i^a\}_{i \in I} : L(G) \to 2^\Sigma$ as follows. For any $s \in L(G)$, the set $\{S_i^a\}_{i \in I}(s)$ of events to be enabled is defined as

$$\{S_i^a\}_{i \in I}(s) \ = \ \Sigma_{uc} \cup \{\sigma \in \Sigma_c | \ [\sigma \in \Sigma_{c,e} \wedge \sigma \notin d(s)] \vee [\sigma \in \Sigma_{c,d} \wedge \sigma \in e(s)]\},$$

where

$$e(s) := \bigcup_{i \in I} e_i(M_i(s)), \ d(s) := \bigcup_{i \in I} d_i(M_i(s)).$$

In the definition of $\{S_i^a\}_{i \in I}(s)$, $e(s) \subseteq \Sigma_c$ is the set of globally enabled events, and $d(s) \subseteq \Sigma_c$ is the set of globally disabled events. For any $s \in L(G)$ and any $\sigma \in \Sigma$, σ is enabled by $\{S_i^a\}_{i \in I}$ if i) it is uncontrollable, ii) it is enabled by default ($\sigma \in \Sigma_{c,e}$) and is not disabled by the "disable" decision ($\sigma \notin d(s)$), or iii) it is disabled by default ($\sigma \in \Sigma_{c,d}$) and is enabled by the "enable" decision ($\sigma \in e(s)$).

[Definition 1] [3] A language $K \subseteq L(G)$ is said to be

- *C&P co-observable* with respect to $A \subseteq \Sigma_c$ if for any $s \in \overline{K}$ and any $\sigma \in A$ with $s\sigma \in L(G) - \overline{K}$, there exists $i \in In(\sigma)$ such that

$$(M_i^{-1} M_i(s) \cap \overline{K})\{\sigma\} \cap \overline{K} = \emptyset.$$

- *D&A co-observable* with respect to $A \subseteq \Sigma_c$ if for any $s \in \overline{K}$ and any $\sigma \in A$ with $s\sigma \in \overline{K}$, there exists $i \in In(\sigma)$ such that

$$(M_i^{-1} M_i(s) \cap \overline{K})\{\sigma\} \cap L(G) \subseteq \overline{K}.$$

[Proposition 1] [3] For a nonempty language $K \subseteq L_m(G)$, there exists a nonblocking decentralized supervisor $\{S_i^a\}_{i \in I} : L(G) \to 2^\Sigma$ such that $L_m(\{S_i^a\}_{i \in I}/G) = K$ under a given partition $\{\Sigma_{c,e}, \Sigma_{c,d}\}$ of Σ_c if and only if K is $L_m(G)$-closed, controllable, C&P co-observable with respect to $\Sigma_{c,e}$, and D&A co-observable with respect to $\Sigma_{c,d}$.

[Definition 2] A language $K \subseteq L(G)$ is said to be *C&P∨D&A co-observable* if there exists a partition $\{\Sigma_{c,e}, \Sigma_{c,d}\}$ of Σ_c such that K is C&P co-observable with respect to $\Sigma_{c,e}$ and D&A co-observable with respect to $\Sigma_{c,d}$.

The Enabling/Disabling architecture has been extended by introducing the *conditional* decisions [4]. This extended architecture is called the conditional Enabling/Disabling architecture. In this extended architecture, each local supervisor S_i^b is defined by a map $S_i^b : M_i(L(G)) \to 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}}$, where

$$S_i^b(M_i(s)) = (e_i(M_i(s)), d_i(M_i(s)), ce_i(M_i(s)), cd_i(M_i(s)))$$

for $s \in L(G)$. The additional components $ce_i(M_i(s))$ and $cd_i(M_i(s))$ represent the following conditional decisions on Σ_{ci}:

$ce_i(M_i(s))$: events that are enabled if no other supervisor disables them,

$cd_i(M_i(s))$: events that are disabled if no other supervisor enables them.

The decentralized supervisor $\{S_i^b\}_{i\in I} : L(G) \to 2^\Sigma$ is defined by a map such that for any $s \in L(G)$,

$$\{S_i^b\}_{i\in I}(s) = \Sigma_{uc} \cup \{\sigma \in \Sigma_c | \ [\sigma \in \Sigma_{c,e} \wedge \sigma \notin d(s)] \vee [\sigma \in \Sigma_{c,d} \wedge \sigma \in e(s)]\},$$

where

$$e(s) := \{\sigma \in \Sigma_c | \ [\sigma \in \bigcup_{i\in I} e_i(M_i(s))] \vee [\sigma \in \bigcup_{i\in I} ce_i(M_i(s)) \wedge \sigma \notin \bigcup_{i\in I} d_i(M_i(s))]\},$$

$$d(s) := \{\sigma \in \Sigma_c | \ [\sigma \in \bigcup_{i\in I} d_i(M_i(s))] \vee [\sigma \in \bigcup_{i\in I} cd_i(M_i(s)) \wedge \sigma \notin \bigcup_{i\in I} e_i(M_i(s))]\}.$$

[Definition 3] [4] For a language $K \subseteq L(G)$, a decentralized supervisor $\{S_i^b\}_{i\in I} : L(G) \to 2^\Sigma$ is said to be *control-nonconflicting* (with respect to K) if for any $s \in \overline{K}$ and any $\sigma \in \Sigma_c$ such that $s\sigma \in L(G)$, $\sigma \notin e(s) \cap d(s)$ holds.

[Definition 4] [4] A language $K \subseteq L(G)$ is said to be

- *conditionally C&P co-observable* with respect to $A \subseteq \Sigma_c$ if for any $s \in \overline{K}$ and any $\sigma \in A$ with $s\sigma \in L(G) - \overline{K}$, there exists $i \in In(\sigma)$ such that

$$(\forall s_i\sigma \in (M_i^{-1}M_i(s) \cap \overline{K})\{\sigma\} \cap \overline{K}) \ \exists j \in In(\sigma); \ (M_j^{-1}M_j(s_i) \cap \overline{K})\{\sigma\} \cap L(G) \subseteq \overline{K}.$$

- *conditionally D&A co-observable* with respect to $A \subseteq \Sigma_c$ if for any $s \in \overline{K}$ and any $\sigma \in A$ with $s\sigma \in \overline{K}$, there exists $i \in In(\sigma)$ such that

$$(\forall s_i\sigma \in (M_i^{-1}M_i(s) \cap \overline{K})\{\sigma\} \cap (L(G) - \overline{K})) \ \exists j \in In(\sigma); \ (M_j^{-1}M_j(s_i) \cap \overline{K})\{\sigma\} \cap \overline{K} = \emptyset.$$

[Proposition 2] [4] For a nonempty language $K \subseteq L_m(G)$, there exists a control-nonconflicting and nonblocking decentralized supervisor $\{S_i^b\}_{i\in I} : L(G) \to 2^\Sigma$ such that $L_m(\{S_i^b\}_{i\in I}/G) = K$ under a given partition $\{\Sigma_{c,e}, \Sigma_{c,d}\}$ of Σ_c if and only if K is $L_m(G)$-closed, controllable, conditionally C&P co-observable with respect to $\Sigma_{c,e}$, and conditionally D&A co-observable with respect to $\Sigma_{c,d}$.

[Definition 5] A language $K \subseteq L(G)$ is said to be *conditionally C&P∨D&A co-observable* if there exists a partition $\{\Sigma_{c,e}, \Sigma_{c,d}\}$ of Σ_c such that K is conditionally C&P co-observable with respect to $\Sigma_{c,e}$ and conditionally D&A co-observable with respect to $\Sigma_{c,d}$.

Note that conditional C&P∨D&A co-observability is weaker than C&P∨D&A co-observability.

3. Decentralized Control with Dynamic Default Control

In this section, we introduce a decentralized control architecture using dynamic default control. Then, we present a necessary and sufficient condition for the existence of a nonblocking decentralized supervisor with dynamic default control.

Each local supervisor S_i^c is defined by a map $S_i^c : M_i(L(G)) \to 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}}$, where

$$S_i^c(M_i(s)) = (e_i(M_i(s)), d_i(M_i(s)), edf_i(M_i(s)), ddf_i(M_i(s)))$$

for $s \in L(G)$. The additional components $edf_i(M_i(s))$ and $ddf_i(M_i(s))$ represent the following control decisions on Σ_{ci}:

$edf_i(M_i(s))$: events that are enabled by default

$ddf_i(M_i(s))$: events that are disabled by default

By using these local decisions, the decentralized supervisor is defined by a map $\{S_i^c\}_{i\in I} : L(G) \to 2^\Sigma$. For any $s \in L(G)$, the set $\{S_i^c\}_{i\in I}(s)$ of events to be enabled is defined as

$$\{S_i^c\}_{i\in I}(s) = \Sigma_{uc} \cup \{\sigma \in \Sigma_c | \ [\sigma \in \Sigma_{c,e}(s) \wedge \sigma \notin d(s)] \vee [\sigma \in \Sigma_{c,d}(s) \wedge \sigma \in e(s)]\},$$

where

$$e(s) := \bigcup_{i\in I} e_i(M_i(s)), \ d(s) := \bigcup_{i\in I} d_i(M_i(s)),$$

$$\Sigma_{c,e}(s) := \bigcup_{i\in I} edf_i(M_i(s)), \ \Sigma_{c,d}(s) := \bigcup_{i\in I} ddf_i(M_i(s)).$$

[Remark 1] The set $\Sigma_{c,e}(s)$ (respectively, $\Sigma_{c,d}(s)$) of events enabled (respectively, disabled) by default is dynamically updated by the local supervisors' decisions $edf_i(M_i(s))$ (respectively, $ddf_i(M_i(s))$).

[Definition 6] For a language $K \subseteq L(G)$, a decentralized supervisor $\{S_i^c\}_{i\in I} : L(G) \to 2^\Sigma$ is said to be *admissible* (with respect to K) if for any $s \in \overline{K}$, $\{\Sigma_{c,e}(s), \Sigma_{c,d}(s)\}$ is a partition of Σ_c.

Intuitively, the admissibility condition implies that for each controllable event, the default control is uniquely determined.

[Definition 7] A language $K \subseteq L(G)$ is said to be *dynami-*

cally C&P∨D&A co-observable if for each $\sigma \in \Sigma_c$, there exists a partition $\{K_e^\sigma, K_d^\sigma\}$ of $\overline{K}$ satisfying the following four conditions:

(1) for any $s \in K_e^\sigma$ with $s\sigma \in L(G) - \overline{K}$, there exists $i \in In(\sigma)$ such that

$$(M_i^{-1}M_i(s) \cap K_e^\sigma)\{\sigma\} \cap \overline{K} = \emptyset.$$

(2) for any $s \in K_d^\sigma$ with $s\sigma \in \overline{K}$, there exists $i \in In(\sigma)$ such that

$$(M_i^{-1}M_i(s) \cap K_d^\sigma)\{\sigma\} \cap L(G) \subseteqq \overline{K}.$$

(3) for any $s \in K_e^\sigma$, there exists $i \in In(\sigma)$ such that $M_i(s) \notin M_i(K_d^\sigma)$.

(4) for any $s \in K_d^\sigma$, there exists $i \in In(\sigma)$ such that $M_i(s) \notin M_i(K_e^\sigma)$.

The following theorem presents necessary and sufficient conditions for the existence of an admissible and nonblocking decentralized supervisor.

[Theorem 1] For a nonempty language $K \subseteqq L_m(G)$, there exists an admissible and nonblocking decentralized supervisor $\{S_i^c\}_{i\in I} : L(G) \to 2^\Sigma$ such that $L_m(\{S_i^c\}_{i\in I}/G) = K$ if and only if K is $L_m(G)$-closed, controllable, and dynamically C&P∨D&A co-observable.

(Proof) (⇒) Suppose that there exists an admissible and nonblocking decentralized supervisor $\{S_i^c\}_{i\in I}$ such that $L_m(\{S_i^c\}_{i\in I}/G) = K$. Then $L_m(G)$-closure and controllability hold. For dynamic C&P∨D&A co-observability, let

$$K_e^\sigma := \{s \in \overline{K} \mid \sigma \in \Sigma_{c,e}(s)\},$$
$$K_d^\sigma := \{s \in \overline{K} \mid \sigma \in \Sigma_{c,d}(s)\}$$

for each $\sigma \in \Sigma_c$. By the admissibility condition of $\{S_i^c\}_{i\in I}$, $\{K_e^\sigma, K_d^\sigma\}$ is a partition of $\overline{K}$. We prove that this partition satisfies the four conditions of dynamic C&P∨D&A co-observability.

(first condition:) Suppose for contradiction that there exists $s \in K_e^\sigma \subseteqq \overline{K}$ with $s\sigma \in L(G) - \overline{K}$ such that for any $i \in In(\sigma)$,

$$(M_j^{-1}M_j(s) \cap K_e^\sigma)\{\sigma\} \cap \overline{K} \neq \emptyset. \quad (1)$$

Since $L(\{S_i^c\}_{i\in I}/G) = \overline{K}$ and $\sigma \in \Sigma_{c,e}(s)$, we have $\sigma \in d(s)$. There exists $j \in In(\sigma)$ such that $\sigma \in d_j(M_j(s))$. From Eq. (1), there exists $s_j \in K_e^\sigma$ such that $s_j\sigma \in (M_j^{-1}M_j(s) \cap K_e^\sigma)\{\sigma\} \cap \overline{K}$ and $\sigma \in d_j(M_j(s_j))$, which implies that $\sigma \in d(s_j)$. Also, since $s_j \in K_e^\sigma$, i.e., $\sigma \in \Sigma_{c,e}(s_j)$, we have $s_j\sigma \notin L(\{S_i^c\}_{i\in I}/G) = \overline{K}$, which is a contradiction.

(second condition:) Suppose for contradiction that there exists $s \in K_d^\sigma \subseteqq \overline{K}$ with $s\sigma \in \overline{K}$ such that for any $i \in In(\sigma)$,

$$(M_j^{-1}M_j(s) \cap K_d^\sigma)\{\sigma\} \cap L(G) \not\subseteqq \overline{K}. \quad (2)$$

Since $\sigma \in \Sigma_{c,d}(s)$, we have $\sigma \in e(s)$. There exists $j \in In(\sigma)$ such that $\sigma \in e_j(M_j(s))$. From Eq. (2), there exists $s_j \in K_d^\sigma$ such that $s_j\sigma \in (M_j^{-1}M_j(s) \cap K_d^\sigma)\{\sigma\} \cap (L(G) - \overline{K})$ and $\sigma \in e_j(M_j(s_j))$, which implies that $\sigma \in e(s_j)$. Also, since $s_j \in K_d^\sigma$, i.e., $\sigma \in \Sigma_{c,d}(s_j)$, we have $s_j\sigma \in L(\{S_i^c\}_{i\in I}/G) = \overline{K}$, which is a contradiction.

(third condition:) For any $s \in K_e^\sigma$, we have $\sigma \in \Sigma_{c,e}(s) = \bigcup_{i\in I} edf_i(M_i(s))$, which implies that there exists $j \in In(\sigma)$ such that $\sigma \in edf_j(M_j(s))$. Then, for any $s_j \in \overline{K} \cap M_j^{-1}M_j(s)$, we have $\sigma \in edf_j(M_j(s_j)) \subseteqq \Sigma_{c,e}(s_j)$, which implies that $s_j \in K_e^\sigma$. It follows that $M_j(s) \notin M_j(K_d^\sigma)$.

(fourth condition:) The fourth condition is proved in the same way as the third condition.

(⇐) We consider a local supervisor $S_i^c : M_i(L(G)) \to 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}}$ such that for any $s \in L(G)$, each component of

$$S_i^c(M_i(s)) = (e_i(M_i(s)), d_i(M_i(s)), edf_i(M_i(s)), ddf_i(M_i(s)))$$

is given as follows:

$$e_i(M_i(s)) := \{\sigma \in \Sigma_{ci} \mid (M_i^{-1}M_i(s) \cap K_d^\sigma)\{\sigma\} \cap L(G) \subseteqq \overline{K})\},$$
$$d_i(M_i(s)) := \{\sigma \in \Sigma_{ci} \mid (M_i^{-1}M_i(s) \cap K_e^\sigma)\{\sigma\} \cap \overline{K} = \emptyset\},$$

$$edf_i(M_i(s)) := \{\sigma \in \Sigma_{ci} \mid M_i(s) \in M_i(K_e^\sigma) - M_i(K_d^\sigma)\},$$
$$ddf_i(M_i(s)) := \{\sigma \in \Sigma_{ci} \mid M_i(s) \in M_i(K_d^\sigma) - M_i(K_e^\sigma)\}.$$

We first show that $\{S_i^c\}_{i\in I}$ is admissible. Suppose for contradiction that there exists $s \in \overline{K}$ such that $\{\Sigma_{c,e}(s), \Sigma_{c,d}(s)\}$ is not a partition of Σ_c. Then, there exists $\sigma \in \Sigma_c$ such that $\sigma \in \Sigma_{c,e}(s) \cap \Sigma_{c,d}(s)$ or $\sigma \notin \Sigma_{c,e}(s) \cup \Sigma_{c,d}(s)$. Let $\sigma \in \Sigma_{c,e}(s) \cap \Sigma_{c,d}(s)$. Since $\sigma \in \bigcup_{i\in I} edf_i(M_i(s))$, there exists $j \in In(\sigma)$ such that $M_j(s) \in M_j(K_e^\sigma) - M_j(K_d^\sigma)$, which implies that $s \notin K_d^\sigma$. Also, since $\sigma \in \bigcup_{i\in I} ddf_i(M_i(s))$, there exists $k \in In(\sigma)$ such that $M_k(s) \in M_k(K_d^\sigma) - M_k(K_e^\sigma)$, which implies that $s \notin K_e^\sigma$. It follows that $s \in \overline{K} - (K_e^\sigma \cup K_d^\sigma)$, which contradicts the assumption that $\{K_e^\sigma, K_d^\sigma\}$ is a partition of $\overline{K}$. Let $\sigma \notin \Sigma_{c,e}(s) \cup \Sigma_{c,d}(s)$. If $s \in K_e^\sigma$, then there exists $j \in In(\sigma)$ such that $M_j(s) \in M_j(K_e^\sigma) - M_j(K_d^\sigma)$, which implies that $\sigma \in edf_j(M_j(s)) \subseteqq \Sigma_{c,e}(s)$. This is a contradiction. If $s \in K_d^\sigma$, then there exists $j \in In(\sigma)$ such that $M_j(s) \in M_j(K_d^\sigma) - M_j(K_e^\sigma)$, which implies that $\sigma \in ddf_j(M_j(s)) \subseteqq \Sigma_{c,d}(s)$. This is also a contradiction.

We next show that $L(\{S_i^c\}_{i\in I}/G) = \overline{K}$ by induction on the length of strings. Since K is nonempty, we have $\varepsilon \in L(\{S_i^c\}_{i\in I}/G) \cap \overline{K}$. We consider any $s \in L(\{S_i^c\}_{i\in I}/G) \cap \overline{K}$ and any $\sigma \in \Sigma$.

Let $s\sigma \in L(\{S_i^c\}_{i\in I}/G) \subseteqq L(G)$. We have the following three cases:

- $\sigma \in \Sigma_{uc}$,
- $\sigma \in \Sigma_{c,e}(s) \wedge \sigma \notin d(s)$,
- $\sigma \in \Sigma_{c,d}(s) \wedge \sigma \in e(s)$.

In the first case, controllability of K implies that $s\sigma \in \overline{K}$.

We consider the second case. Suppose for contradiction that $s\sigma \notin \overline{K}$. Since $\sigma \in \Sigma_{c,e}(s)$, there exists $i \in In(\sigma)$ such that $\sigma \in edf_i(M_i(s))$. We have $M_i(s) \in M_i(K_e^\sigma) - M_i(K_d^\sigma)$, which implies that $s \in K_e^\sigma$. By the first condition of dynamic C&P∨D&A co-observability of K, there exists $j \in In(\sigma)$ such that

$$(M_j^{-1}M_j(s) \cap K_e^\sigma)\{\sigma\} \cap \overline{K} = \emptyset,$$

which implies that $\sigma \in d_j(M_j(s)) \subseteq d(s)$. This is a contradiction.

We consider the third case. Suppose for contradiction that $s\sigma \notin \overline{K}$. Since $\sigma \in \Sigma_{c,d}(s)$, there exists $i \in In(\sigma)$ such that $\sigma \in ddf_i(M_i(s))$, which implies that $s \in K_d^\sigma$. By $s \in K_d^\sigma$ and $s\sigma \in L(G) - \overline{K}$, we have $\sigma \notin \bigcup_{i\in I} e_i(M_i(s)) = e(s)$, which is a contradiction.

Next, let $s\sigma \in \overline{K} \subseteq L(G)$. Suppose for contradiction that $s\sigma \notin L(\{S_i^c\}_{i\in I}/G)$ i.e., $\sigma \notin \{S_i^c\}_{i\in I}(s)$. Since $\{\Sigma_{c,e}(s), \Sigma_{c,d}(s)\}$ is a partition of Σ_c, we have the following two cases:

- $\sigma \in \Sigma_{c,e}(s) \wedge \sigma \in d(s)$,
- $\sigma \in \Sigma_{c,d}(s) \wedge \sigma \notin e(s)$.

We consider the first case. Since $\sigma \in \Sigma_{c,e}(s)$, we have $s \in K_e^\sigma$. By $s \in K_e^\sigma$ and $s\sigma \in \overline{K}$, we have $\sigma \notin \bigcup_{i\in I} d_i(M_i(s)) = d(s)$, which is a contradiction.

We consider the second case. Since $\sigma \in \Sigma_{c,d}(s)$, we have $s \in K_d^\sigma$. By the second condition of dynamic C&P∨D&A co-observability of K, there exists $j \in In(\sigma)$ such that

$$(M_j^{-1}M_j(s) \cap K_d^\sigma)\{\sigma\} \cap L(G) \subseteq \overline{K},$$

which implies that $\sigma \in e_j(M_j(s)) \subseteq e(s)$. This is a contradiction.

Therefore, we have $L(\{S_i^c\}_{i\in I}/G) = \overline{K}$. Since K is $L_m(G)$-closed, $\{S_i^c\}_{i\in I}$ is an admissible and nonblocking decentralized supervisor such that $L_m(\{S_i^c\}_{i\in I}/G) = K$. □

Note that dynamic C&P∨D&A co-observability is also weaker than C&P∨D&A co-observability. In the following, we present an example of a language that is dynamically C&P∨D&A co-observable, but not conditionally C&P∨D&A co-observable.

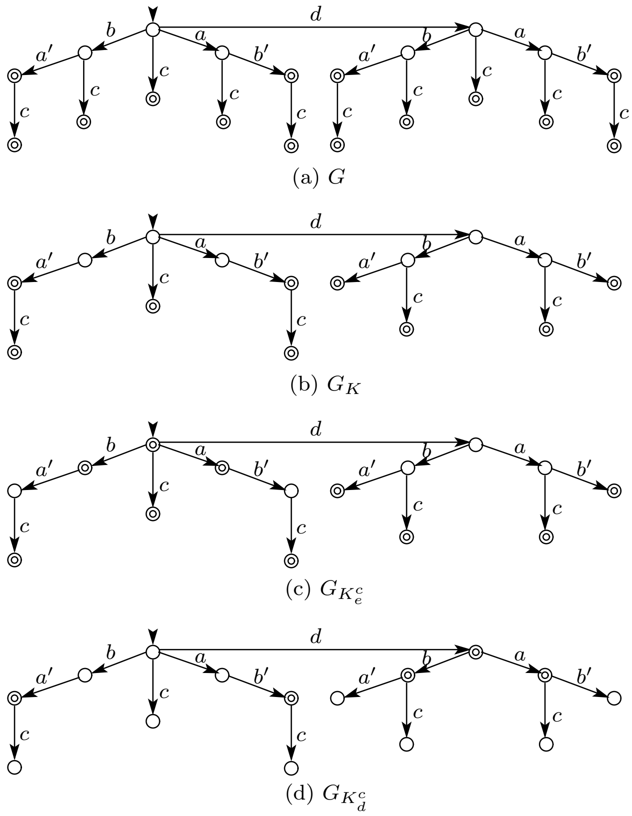
[Example 1] We consider a DES modeled by the automaton G shown in Fig. 1 (a), which is a modified version of the DES considered in [4]. A double circle is used to identify a marked state. Let $n = 2$, $\Sigma_{1c} = \Sigma_{2c} = \{c\}$,

$$M_1(\sigma) = \begin{cases} \sigma & \text{if } \sigma \in \{a, a', c, d\}, \\ \varepsilon & \text{otherwise,} \end{cases}$$

and

$$M_2(\sigma) = \begin{cases} \sigma & \text{if } \sigma \in \{b, b', c, d\}, \\ \varepsilon & \text{otherwise.} \end{cases}$$

Also, let $K \subseteq L(G)$ be a language marked by the automaton G_K shown in Fig. 1 (b). Clearly, K is $L_m(G)$-closed and controllable.



(a) G (b) G_K (c) $G_{K_e^c}$ (d) $G_{K_d^c}$

Fig. 1 Automata G, G_K, $G_{K_e^c}$, and $G_{K_d^c}$ of Example 1.

We first show that K is not conditionally C&P∨D&A co-observable. We consider $dc \in L(G) - \overline{K}$ where $d \in \overline{K}$ and $c \in \Sigma_c$. We have $dbc \in (M_1^{-1}M_1(d) \cap \overline{K})\{c\} \cap \overline{K}$, $dc \in (M_1^{-1}M_1(db) \cap \overline{K})\{c\} \cap (L(G) - \overline{K})$, and $dba'c \in (M_2^{-1}M_2(db) \cap \overline{K})\{c\} \cap (L(G) - \overline{K})$. Also, we have $dac \in (M_2^{-1}M_2(d) \cap \overline{K})\{c\} \cap \overline{K}$, $dab'c \in (M_1^{-1}M_1(da) \cap \overline{K})\{c\} \cap (L(G) - \overline{K})$, and $dc \in (M_2^{-1}M_2(da) \cap \overline{K})\{c\} \cap (L(G) - \overline{K})$. Therefore, K is not conditionally C&P co-observable with respect to $\{c\}$. We then consider $c \in \overline{K}$ where $\varepsilon \in \overline{K}$ and $c \in \Sigma_c$. We have $bc \in (M_1^{-1}M_1(\varepsilon) \cap \overline{K})\{c\} \cap (L(G) - \overline{K})$, $c \in (M_1^{-1}M_1(b) \cap \overline{K})\{c\} \cap \overline{K}$, and $ba'c \in (M_2^{-1}M_2(b) \cap \overline{K})\{c\} \cap \overline{K}$. Also, we have $ac \in (M_2^{-1}M_2(\varepsilon) \cap \overline{K})\{c\} \cap (L(G) - \overline{K})$, $ab'c \in (M_1^{-1}M_1(a) \cap \overline{K})\{c\} \cap \overline{K}$, and $c \in (M_2^{-1}M_2(a) \cap \overline{K})\{c\} \cap \overline{K}$. Therefore, K is not conditionally D&A co-observable with respect to $\{c\}$. It follows that K is not conditionally C&P∨D&A co-observable. Thus, K cannot be achieved in the conditional Enabling/Disabling architecture of [4].

We next show that K is dynamically C&P∨D&A co-observable. For the controllable event c, we consider a par-

tition $\{K_e^c, K_d^c\}$ of $\overline{K}$, where K_e^c and K_d^c are the marked languages of $G_{K_e^c}$ and $G_{K_d^c}$ shown in Fig. 1 (c) and (d), respectively. Then, the four conditions of dynamical C&P∨D&A co-observability are examined as follows:
(First Condition) Since

$$a \in K_e^c,\ ac \in L(G) - \overline{K},\ (M_1^{-1}M_1(a) \cap K_e^c)\{c\} \cap \overline{K} = \emptyset,$$

$$b \in K_e^c,\ bc \in L(G) - \overline{K},\ (M_2^{-1}M_2(b) \cap K_e^c)\{c\} \cap \overline{K} = \emptyset,$$

$$dba' \in K_e^c,\ dba'c \in L(G) - \overline{K},$$
$$(M_1^{-1}M_1(dba') \cap K_e^c)\{c\} \cap \overline{K} = \emptyset,$$

$$dab' \in K_e^c,\ dab'c \in L(G) - \overline{K},$$
$$(M_2^{-1}M_2(dab') \cap K_e^c)\{c\} \cap \overline{K} = \emptyset,$$

the first condition holds.
(Second Condition) Since

$$ba' \in K_d^c,\ ba'c \in \overline{K},\ (M_1^{-1}M_1(ba') \cap K_d^c)\{c\} \cap L(G) \subseteq \overline{K},$$

$$ab' \in K_d^c,\ ab'c \in \overline{K},\ (M_2^{-1}M_2(ab') \cap K_d^c)\{c\} \cap L(G) \subseteq \overline{K},$$

$$da \in K_d^c,\ dac \in \overline{K},\ (M_1^{-1}M_1(da) \cap K_d^c)\{c\} \cap L(G) \subseteq \overline{K},$$

$$db \in K_d^c,\ dbc \in \overline{K},\ (M_2^{-1}M_2(db) \cap K_d^c)\{c\} \cap L(G) \subseteq \overline{K},$$

the second condition holds.
(Third Condition) Since

$$M_1(K_d^c) = \{a, a', d, da\},\ M_2(K_d^c) = \{b, b', d, db\},$$

$$\varepsilon, b \in K_e^c,\ M_1(\varepsilon) = M_1(b) = \varepsilon \notin M_1(K_d^c),$$

$$a \in K_e^c,\ M_2(a) = \varepsilon \notin M_2(K_d^c),$$

$$c \in K_e^c,\ M_1(c) = c \notin M_1(K_d^c),$$

$$ab'c \in K_e^c,\ M_1(ab'c) = ac \notin M_1(K_d^c),$$

$$ba'c \in K_e^c,\ M_1(ba'c) = a'c \notin M_1(K_d^c),$$

$$dab' \in K_e^c,\ M_2(dab') = db' \notin M_2(K_d^c),$$

$$dac \in K_e^c,\ M_1(dac) = dac \notin M_1(K_d^c),$$

$$dba' \in K_e^c,\ M_1(dba') = da' \notin M_1(K_d^c),$$

$$dbc \in K_e^c,\ M_1(dbc) = dc \notin M_1(K_d^c),$$

the third condition holds.
(Fourth Condition) Since

$$M_1(K_e^c) = \{\varepsilon, a, c, ac, a'c, da, da', dc, dac\},$$
$$M_2(K_e^c) = \{\varepsilon, b, c, bc, b'c, db, db', dc, dbc\},$$

$$ab' \in K_d^c,\ M_2(ab') = b' \notin M_2(K_e^c),$$

$$ba' \in K_d^c,\ M_1(ba') = a' \notin M_1(K_e^c),$$

$$d, db \in K_d^c,\ M_1(d) = M_1(db) = d \notin M_1(K_e^c),$$

$$da \in K_d^c,\ M_2(da) = d \notin M_2(K_e^c),$$

the fourth condition holds.

We construct two local supervisors $S_i^c : M_i(L(G)) \to 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}} \times 2^{\Sigma_{ci}}$ $(i = 1, 2)$ as follows:

$$S_1^c(t) = \begin{cases} (\{c\}, \emptyset, \{c\}, \emptyset) & \text{if } t = \varepsilon, \\ (\emptyset, \{c\}, \emptyset, \{c\}) & \text{if } t = d, \\ (\{c\}, \{c\}, \emptyset, \emptyset) & \text{if } t \in \{a, da, da'c\}, \\ (\{c\}, \{c\}, \emptyset, \{c\}) & \text{if } t = a', \\ (\{c\}, \{c\}, \{c\}, \emptyset) & \text{if } t \in \{c, ac, a'c, da', dc, dac\}, \end{cases}$$

$$S_2^c(t) = \begin{cases} (\{c\}, \emptyset, \{c\}, \emptyset) & \text{if } t = \varepsilon, \\ (\emptyset, \{c\}, \emptyset, \{c\}) & \text{if } t = d, \\ (\{c\}, \{c\}, \emptyset, \emptyset) & \text{if } t \in \{b, db, db'c\}, \\ (\{c\}, \{c\}, \emptyset, \{c\}) & \text{if } t = b', \\ (\{c\}, \{c\}, \{c\}, \emptyset) & \text{if } t \in \{c, bc, b'c, db', dc, dbc\}. \end{cases}$$

By the proof of Theorem 1, the decentralized supervisor $\{S_i^c\}_{i \in I} : L(G) \to 2^{\Sigma}$ constructed above is an admissible and nonblocking one such that $L_m(\{S_i^c\}_{i \in I}/G) = K$.

4. Conclusion

We have studied decentralized supervisory control of DESs using dynamic default control. We have introduced a notion of dynamic C&P∨D&A co-observability, and proved that $L_m(G)$-closure, controllability, and dynamic C&P∨D&A co-observability serve as necessary and sufficient conditions for the existence of a decentralized supervisor with dynamic default control. We are currently investigating how to verify dynamic C&P∨D&A co-observability.

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References

[1] R. Cieslak, C. Desclaux, A. S. Fawaz, and P. Varaiya, "Supervisory control of discrete-event processes with partial observations," IEEE Trans. Automat. Contr., vol.33, no.3, pp.249–260, 1988.

[2] K. Rudie and W. M. Wonham, "Think globally, act locally: decentralized supervisory control," IEEE Trans. Automat. Contr., vol.37, no.11, pp.1692–1708, 1992.

[3] T.-S. Yoo and S. Lafortune, "A general architecture for decentralized supervisory control of discrete-event systems," Discrete Event Dynamic Syst.: Theory and Appl., vol.12, no.3, pp.335–377, 2002.

[4] T.-S. Yoo and S. Lafortune, "Decentralized supervisory control with conditional decisions: Supervisor existence", IEEE Trans. Automat. Contr., vol.49, no.11, pp.1886–1904, 2004.

[5] P. J. Ramadge and W. M. Wonham, "Supervisory control of a class of discrete-event processes," SIAM J. Contr. Optim., vol.25, no.1, pp.206–230, 1987.