MLD Form Based Design of Piecewise Affine Systems with Specified Symbolic Dynamics

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***Abstract*—In this paper, we focus on piecewise affine(PWA) systems. In PWA systems, the state space is partitioned into several regions and in each region, system behavior is described by an affine equation. We will call each region a mode. It is well-known that PWA systems can be expressed by mixed logical dynamical(MLD) systems. We will propose a design method for PWA systems with specified symbolic dynamics, where specified periodic mode transitions exhibit. Constant terms in PWA systems are taken as design parameters, and by determining configuration of an equilibrium of the affine equation in each mode, we construct PWA systems satisfying the specifications. We will apply the proposed method to the design of a continuous-time switching network as an example.**

I. Introduction

A piecewise affine (abbr. PWA) system is often used as a model of hybrid dynamical systems[1], [2], which contain both continuous and logical components. Moreover, piecewise affine approximation is applied to a nonlinear system in order to simplify analysis of the system. Therefore, analysis and control of PWA systems have been paid much attention to[3].

In this paper, we focus on controlling symbolic dynamics in a given PWA systems. By coupling another PWA system, more interactions rise between the original system and the added one. We control the original system in order to have a given specified symbolic dynamics by adequate design of the interactions.

On the other hand, Bemporad and Morari proposed mixed logical dynamical (abbr. MLD) systems[4], which can express a class of hybrid dynamical systems, including PWA systems. The MLD forms contain inequalities caused by propositional calculus and linear integer programming[5].

In this paper, we will propose a novel control method by applying a concept of the MLD forms to the control problem, and show that the control problem can be expressed by an optimization problem with linear constraints inequalities derived by the MLD forms. We will also apply the proposed method to controlling a continuous-time switching network[6], which is a model of gene networks, as an example.

This paper is organized as follows: In Sec. I, we consider a class of PWA systems and their dynamics. In Sec. II, we explain MLD forms, and show transformation from PWA systems to MLD forms. In Sec. III, we explain controlling symbolic dynamics of a given PWA system by adding another PWA one. In Sec. IV, we propose a novel method to control the symbolic dynamics. In Sec. V, we show an application of the proposed method to controlling a gene network. Finally, in Sec. VI, we conclude this paper.

II. PWA Systems and Their Dynamics

A. PWA systems

We consider an n-dimensional PWA system, and denote $N = \{1, \ldots, n\}$. In this paper, we assume that the system can be described by the following diagonal form:

$$\dot{x} = -Kx + \tilde{F}(y), \tag{1}$$

where $x = [x_1, \ldots, x_n]^{\mathrm{T}}$ is a state vector, $K = \mathrm{diag}[k_1, \ldots, k_n]$ is a damping and each $k_i > 0$, $y = [y_1, \ldots, y_n]^{\mathrm{T}}$ called *mode* is a binary vector determined by x, and $\tilde{F}(y) = [\tilde{F}_1(y), \ldots, \tilde{F}_n(y)]^{\mathrm{T}}$ is an interaction term determined by y. Each $y_i (i \in N)$ of y is a binary variable and defined by

$$y_i = g(x_i) = \begin{cases} 0 & \text{if } x_i < 0, \\ 1 & \text{if } x_i \geq 0. \end{cases} \tag{2}$$

Each $\tilde{F}_i(y)$ is given by

$$\tilde{F}_i(y) = \sum_{i_0 \in 2^N} \left(w_{i_0}^{(i)} \prod_{j \in i_0} y_j \right), \tag{3}$$

where the power set 2^N of N is an index set for the summation and $\prod_{j \in \emptyset} y_j = 1$ for the empty set.

B. Dynamics in the PWA Systems

Corresponding to each mode $(y_1, \ldots, y_n)$, we partition a state space into $R_{y_1 \cdots y_n} = \{(x_1, \ldots, x_n) \mid y_1 = g(x_1), \ldots, y_n = g(x_n)\}$. We define candidates of equilibrium points $e_{y_1 \cdots y_n} = (x_1^*, \ldots, x_n^*)$ for a fixed $(y_1, \ldots, y_n)$, where $x_i^* = \tilde{F}_i(y)/k_i$. If $e_{y_1 \cdots y_n} \in R_{y_1 \cdots y_n}$, it is a stable equilibrium point(abbr. SEP) in $R_{y_1 \cdots y_n}$. On the other hand, if $e_{y_1 \cdots y_n} \notin R_{y_1 \cdots y_n}$, it does not exist, and it is called "an virtual equilibrium point"(abbr. VEP).

Figure 1 shows an example of a 2-dimensional PWA system. In the figure, black and white circles denote SEPs and VEPs, respectively. If an initial condition lies in R_{11}, and its orbit heads toward $e_{11} \in R_{10}$, which is a VEP. However, when the orbit hits the positive x_1 axis in the phase plane, the mode of

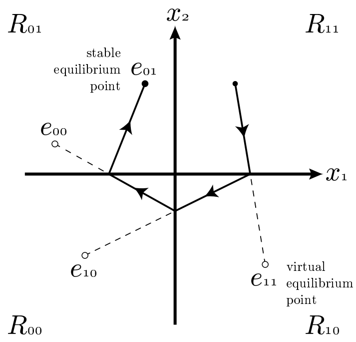


Fig. 1. Dynamics in PWA systems.

the PWA system changes from $(1,1)$ to $(1,0)$, then it heads toward $e_{10} \in R_{00}$, which is also a VEP. Finally, the orbit converges at the SEP e_{01}.

III. MLD Forms of PWA Systems

A. MLD forms

Hybrid dynamical systems are systems such that contain both continuous and logical components. PWA systems are a class of hybrid dynamical systems. In Bemporad and Morari[4], a class of hybrid dynamical systems, including PWA systems, has been introduced in which logic, dynamics, and constraints are integrated, which is called *MLD systems* or *MLD forms*. MLD systems are generally expressed by

$$\begin{cases} \dot{x} = Ax(t) + B_1u(t) + B_2\delta(t) + B_3z(t), \\ y(t) = Cx(t) + D_1u(t) + D_2\delta(t) + D_3z(t), \\ E_2\delta(t) + E_3z(t) \leq E_1u(t) + E_4z(t) + E_5, \end{cases} \tag{4}$$

where $t \in R$, $x = [x_c^{\mathrm{T}}\ x_\ell^{\mathrm{T}}]^{\mathrm{T}} \in R^{n_c} \times \{0,1\}^{n_\ell}$ is the state of the system, $y = [y_c^{\mathrm{T}}\ y_\ell^{\mathrm{T}}]^{\mathrm{T}} \in R^{p_c} \times \{0,1\}^{p_\ell}$ is the output vector, $u = [u_c^{\mathrm{T}}\ u_\ell^{\mathrm{T}}]^{\mathrm{T}} \in R^{m_c} \times \{0,1\}^{m_\ell}$ is the input, and $z \in R^{r_r}$ and $\delta \in \{0,1\}^{r_b}$ is auxiliary vectors. Note that δ and z are used in order to express logical dynamics. The inequalities in Eq. (4) have to be interpreted componentwise, and they are introduced by propositional calculus and linear integer programming[5].

B. Transformation from PWA systems to MLD forms

We define a logical variable $\delta_{\{i\}}$ as $[\delta_{\{i\}} = 1] \leftrightarrow [x_i \geq 0]$, then $\delta_{\{i\}}$ is expressed by the following set of inequalities:

$$\begin{cases} -m\delta_{\{i\}} \leq x_i - m, \\ -(M+\varepsilon)\delta_{\{i\}} \leq -x_i - \varepsilon, \end{cases} \tag{5}$$

where m and M are defined by

$$m = \min_{i\in N,\ t\in R} x_i(t),\ M = \max_{i\in N,\ t\in R} x_i(t). \tag{6}$$

Then, we focus on a product of logical variables in (3). The product term $\delta_{\{i,j\}} = \delta_{\{i\}}\delta_{\{j\}}$ $(i,j \in N)$ can be expressed by the following linear constraints:

$$\begin{cases} -\delta_{\{i\}} + \delta_{\{i,j\}} \leq 0, \\ -\delta_{\{j\}} + \delta_{\{i,j\}} \leq 0, \\ \delta_{\{i\}} + \delta_{\{j\}} - \delta_{\{i,j\}} \leq 1. \end{cases} \tag{7}$$

By applying Eq. (7) repeatedly, Eq. (3) can be linearized about $\delta = \{\delta_{i_0} | i_0 \in 2^N\}$, and expressed by

$$\tilde{F}_i(y) = F_i(\delta) = \sum_{i_0\in 2^N} w_{i_0}^{(i)}\delta_{i_0}. \tag{8}$$

Finally, for a given PWA system (1), the following form is derived:

$$\dot{x} = -Kx + F(\delta), \tag{9}$$

where $F(\delta) = [F_1(\delta), \ldots, F_n(\delta)]^{\mathrm{T}}$. Since Eq. (8) shows that each $F_i(\delta)$ is a weighted linear summation with respect to δ, the term $F(\delta)$ in Eq. (9) can be expressed by the product between a weighted matrix B_2 and 2^n-dimensional vector δ. Therefore, Eq. (9) is the first equation of the MLD form (4). On the other hand, the third inequalities in Eq. (4), which are constraint conditions of an MLD forms, are derived from Eqs. (5) and (7).

IV. What Is "Control"?

Consider the PWA system (1). The system's behavior is deterministic because all the parameters k_i and $w_{i_0}^{(i)}$ $(i \in N, i_0 \in 2^N)$ are given. Therefore, its mode transition sequence is also deterministic.

Consider another PWA system and add it into the original one. We call the given n_o-dimensional PWA system *the original PWA system*, we assume that *the added PWA system* is n_c-dimensional, and we define $N_o = \{1, \ldots, n_o\}$, $N_c = \{n_o + 1, \ldots, n_o + n_c\}$, and $N = N_o \cup N_c$. Adding a PWA system makes more interaction terms. For example, for $i \in N_o$,

$$\begin{aligned} F_i &= \sum_{i_0\in 2^{N_o}} \sum_{j_0\in 2^{N_c}} w_{i_0\cup j_0}^{(i)}\delta_{i_0}\delta_{j_0} \\ &= \sum_{i_0\in 2^{N_o}} w_{i_0}^{(i)}\delta_{i_0} + \sum_{i_0\in 2^{N_o}} \sum_{j_0\in 2^{N_c}\setminus\emptyset} w_{i_0\cup j_0}^{(i)}\delta_{i_0}\delta_{j_0}. \end{aligned} \tag{10}$$

By adding another PWA system, the second term in Eq. (10) rises but the first term still remains. We call the first and the second term in Eq. (10) a *self* and a *mutual interaction term*, respectively. By using these interaction terms, we can express the coupled system as follows:

$$\begin{bmatrix} \dot{x}_o \\ \dot{x}_c \end{bmatrix} = -\begin{bmatrix} K_o & 0 \\ 0 & K_c \end{bmatrix}\begin{bmatrix} x_o \\ x_c \end{bmatrix} + \begin{bmatrix} F_o(y_o) \\ F_c(y_c) \end{bmatrix} + \begin{bmatrix} U_o(y_o, y_c) \\ Z_c(y_c, y_o) \end{bmatrix}, \tag{11}$$

where $x_o = [x_1, \ldots, x_{n_o}]^{\mathrm{T}}$ and $x_c = [x_{n_o+1}, \ldots, x_{n_o+n_c}]^{\mathrm{T}}$ are state vectors of the original and the added PWA systems, respectively, $K_o = \mathrm{diag}[k_1, \ldots, k_{n_o}]$ and $K_c = \mathrm{diag}[k_{n_o+1}, \ldots, k_{n_o+n_c}]$ are their dampings, $y_o = [y_1, \ldots, y_{n_o}]^{\mathrm{T}}$ and $y_c = [y_{n_o+1}, \ldots, y_{n_o+n_c}]^{\mathrm{T}}$ are their logical vectors, $F_o = [F_1, \ldots, F_{n_o}]^{\mathrm{T}}$ and $F_c = [F_{n_o+1}, \ldots, F_{n_o+n_c}]^{\mathrm{T}}$ are self interaction vectors of them, $U_o = [U_1, \ldots, U_{n_o}]^{\mathrm{T}}$ is a mutual interaction vector from the coupled system to the original one, and $Z_c = [Z_{n_o+1}, \ldots, Z_{n_o+n_c}]^{\mathrm{T}}$ is a mutual interaction vector from the coupled one to the added one. For $i \in N_o$ and $j \in N_c$, F_j, U_i and Z_j are given as follows:

$$F_j = \sum_{j_0\in 2^{N_c}} w_{j_0}^{(j)}\delta_{j_0}, \tag{12}$$

$$U_i = \sum_{i_0\in 2^{N_o}} \sum_{j_0\in 2^{N_c}\setminus\emptyset} w_{i_0\cup j_0}^{(i)}\delta_{i_0}\delta_{j_0}, \tag{13}$$

$$Z_j = \sum_{i_0\in 2^{N_o}\setminus\emptyset} \sum_{j_0\in 2^{N_c}} w_{i_0\cup j_0}^{(j)}\delta_{i_0}\delta_{j_0}. \tag{14}$$

In this paper, *control* means that the mode transition sequence in the original PWA system changes into a specified one by coupling with an added PWA system. In other words, we control the sequence in the original PWA system by giving appropriate U_o, Z_c, and F_c.

V. Design of Specified Symbolic Dynamics

A. Mixed integer inequality form based placement of VEPs

In this subsection, we consider a general n-dimensional PWA system (1).

The example in Sec. II.B shows that it is important where to place SEPs in order to realize a specified mode transitions. That is, if a transition from $(y_1, \ldots, y_i, \ldots, y_n)$ to $(y_1, \ldots, \bar{y}_i, \ldots, y_n)$ is specified, where $\bar{y}_i$ is reverse of y_i, then we set a VEP $e_{y_1\ldots y_i\ldots y_n}$ such that the following placement holds:

$$e_{y_1\ldots y_i\ldots y_n} \in R_{y_1\ldots\bar{y}_i\ldots y_n}. \tag{15}$$

Note that the system's behavior converges to a border of the region $R_{y_1\ldots\bar{y}_i\ldots y_n}$ if the VEP is placed on the border. In order to avoid this placement, we define the following forbidden region B where VEPs are not allowed to be placed.

$$B = \{(x_1, \ldots, x_n) \mid -b < x_i < b \ \ (\forall i \in N)\}, \tag{16}$$

where b is positive. Then, we set the VEP $e_{y_1\ldots y_i\ldots y_n}$ satisfying the following region:

$$e_{y_1\ldots y_i\ldots y_n} \in R_{y_1\ldots\bar{y}_i\ldots y_n}\setminus B. \tag{17}$$

When the i-th element is reversed like Eq. (17), the following relation holds:

$$\begin{cases} x_i^* = F_i/k_i \geq b & \text{if} \quad \delta_{\{i\}} = y_i = 0, \\ x_i^* = F_i/k_i \leq -b & \text{if} \quad \delta_{\{i\}} = y_i = 1, \end{cases} \tag{18}$$

and for each $j \neq i$,

$$\begin{cases} x_j^* = F_j/k_j \leq -b & \text{if} \quad \delta_{\{j\}} = y_j = 0, \\ x_j^* = F_j/k_j \geq b & \text{if} \quad \delta_{\{j\}} = y_j = 1. \end{cases} \tag{19}$$

Since $\delta_{\{i\}}$ and $\delta_{\{j\}}$ are binary variables, the relations (18) and (19) can be reduced to

$$\begin{cases} \text{the reversed element } i & \left(2\delta_{\{i\}} - 1\right) x_i'^* \leq -b, \\ \text{the other elements } j \in N \backslash \{i\} & \left(2\delta_{\{j\}} - 1\right) x_j'^* \geq b. \end{cases} \tag{20}$$

By using Eq. (20), we will formulate the control problem in the following subsection.

B. Formulation of the control problem

We will formulate controlling the coupled system. In this paper, for simplicity, we assume $n_c = 1$, that is, we will control an n_o-dimensional PWA system by adding a one-dimensional PWA system. We show configuration of the following mode transition t by using MLD forms:

$$(y_1, \ldots, y_i, \ldots, y_{n_o}) \xrightarrow{t} (y_1, \ldots, \bar{y}_i, \ldots, y_{n_o}). \tag{21}$$

Note that the specification for y_{n_o+1} is not given, where y_{n_o+1} is a mode of the added element. Therefore, it is possible for y_{n_o+1} to reverse before the change of y_i. Consequently, the mode transition t can be decomposed by the following two consecutive transitions t_1 and t_2:

$$\begin{aligned} &(y_1, \ldots, y_i, \ldots, y_{n_o}, y_{n_o+1}) \\ &\quad \xrightarrow{t_1} (y_1, \ldots, y_i, \ldots, y_{n_o}, y'_{n_o+1}) \\ &\qquad \xrightarrow{t_2} (y_1, \ldots, \bar{y}_i, \ldots, y_{n_o}, y'_{n_o+1}), \end{aligned} \tag{22}$$

where y'_{n_o+1} is the added element's mode after the transition t. The transition t_1 is a transition such that the added element is reversed before the i-the element is reversed, and the transition t_2 is a transition such that the element i is reversed. If $y_{n_o+1} = y'_{n_o+1}$, we do not need to take the transition t_1 into consideration, and the designed mode transition t is directly realized by the following placement:

$$e_{y_1 \cdots y_i \cdots y_{n_o} y_{n_o+1}} \in R_{y_1 \cdots \bar{y}_i \cdots y_{n_o} y_{n_o+1}} \backslash B. \tag{23}$$

On the other hand, if $y_{n_o+1} \neq y'_{n_o+1}$, the mode transition (21) is realized by the both transitions t_1 and t_2. In this case, VEPs must be placed as follows:

$$\begin{cases} e_{y_1 \cdots y_i \cdots y_{n_o} y_{n_o+1}} \in R_{y_1 \cdots y_i \cdots y_{n_o} y'_{n_o+1}} \backslash B, \\ e_{y_1 \cdots y_i \cdots y_{n_o} y'_{n_o+1}} \in R_{y_1 \cdots \bar{y}_i \cdots y_{n_o} y'_{n_o+1}} \backslash B. \end{cases} \tag{24}$$

We introduce the following binary variable ζ:

$$[\zeta = 1] \leftrightarrow \left[\delta_{\{n_o+1\}} = \delta'_{\{n_o+1\}}\right]. \tag{25}$$

Note that for any $i' \in N = N_o \cup N_c$, $\delta_{\{i'\}} = y_{i'}$. Equation (25) can be rewritten as follows:

$$\begin{cases} [\delta_{\{n_o+1\}} = 0] \wedge [\delta'_{\{n_o+1\}} = 0] \rightarrow [\zeta = 1], \\ [\delta_{\{n_o+1\}} = 1] \wedge [\delta'_{\{n_o+1\}} = 0] \rightarrow [\zeta = 0], \\ [\delta_{\{n_o+1\}} = 0] \wedge [\delta'_{\{n_o+1\}} = 1] \rightarrow [\zeta = 0], \\ [\delta_{\{n_o+1\}} = 1] \wedge [\delta'_{\{n_o+1\}} = 1] \rightarrow [\zeta = 1]. \end{cases} \tag{26}$$

Thus, the following mixed-integer inequalities are obtained by applying MLD forms to Eq. (26).

$$\begin{cases} -\delta_{\{n_o+1\}} - \delta'_{\{n_o+1\}} - \zeta \leq -1, \\ \delta_{\{n_o+1\}} - \delta'_{\{n_o+1\}} + \zeta \leq 1, \\ -\delta_{\{n_o+1\}} + \delta'_{\{n_o+1\}} + \zeta \leq 1, \\ \delta_{\{n_o+1\}} + \delta'_{\{n_o+1\}} - \zeta \leq 1. \end{cases} \tag{27}$$

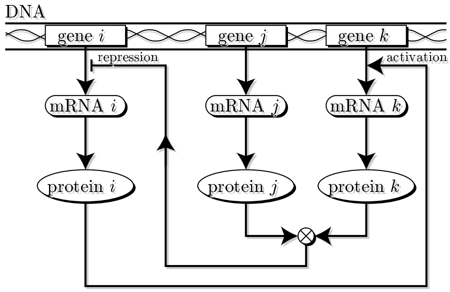


Fig. 2. A gene network.

If $\zeta = 0$, the transition t_1 is needed. From Eq. (20), it is realized by

$$\begin{cases} \left[\bar{\zeta} = 1\right] \rightarrow \left[(2\delta_{\{j\}} - 1)x_j^* \geq b\right], \\ \left[\bar{\zeta} = 1\right] \rightarrow \left[(2\delta_{\{n_o+1\}} - 1)x_{n_o+1}^* \leq -b\right], \end{cases} \tag{28}$$

where each $j \in N_o$. Equation (28) can be expressed by the following inequalities:

$$\begin{cases} -\left(2\delta_{\{j\}} - 1\right) x_j^* + b \leq M\zeta, \\ \left(2\delta_{\{n_o+1\}} - 1\right) x_{n_o+1}^* + b \leq M\zeta. \end{cases} \tag{29}$$

However, Eq. (29) is nonlinear with respect to both logical and continuous variables. In order to linearize Eq. (29), for each $i' \in N$, we define $s_{i'} = \delta_{\{i'\}} x_{i'}^*$ with the following constraints:

$$\begin{cases} s_{i'} \geq m\delta_{\{i'\}}, \; s_{i'} \leq x_{i'}^* - m\left(1 - \delta_{\{i'\}}\right), \\ s_{i'} \leq M\delta_{\{i'\}}, \; s_{i'} \geq x_{i'}^* - M\left(1 - \delta_{\{i'\}}\right). \end{cases} \tag{30}$$

By using $s_{i'}$, Eq. (29) can be linearized.

Then, we consider the mode transition t_2. Note that the transition t_2 must be realized whichever ζ is 0 or 1. For each $j \in N_o \backslash \{i\}$, the transition t_2 can be expressed by

$$\begin{cases} (2\delta_{\{j\}} - 1)x_j^* \geq b, \\ (2\delta_{\{i\}} - 1)x_i^* \leq -b, \\ (2\delta'^{*}_{\{n_o+1\}} - 1)x'^{*}_{n_o+1} \geq b. \end{cases} \tag{31}$$

We can linearize Eq. (31) by the same way in the mode transition t_1.

Therefore, all constraints can be linearized by using auxiliary variables. If we give a suitable performance function J, the control problem reduces to mixed-integer linear programming with linear constraints.

VI. An Application to A Gene Network

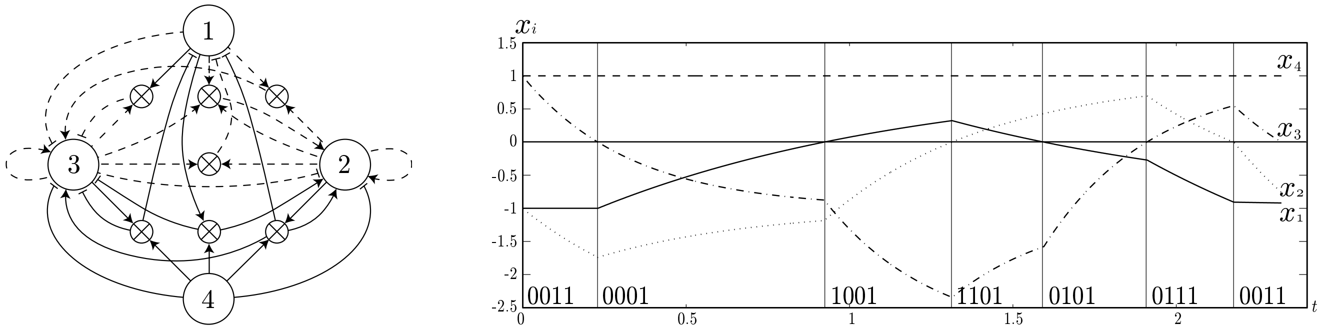
A. DNAs, mRNAs, proteins, and protein interactions

Central dogma is an important concept which explains roles of genes. As shown in Fig. 2, each DNA is transcribed to an mRNA, and each mRNA is translated to a protein, which plays substantial vital role in living organisms.

It is known that there exists feedback mechanism in gene networks. For example, Fig. 2 shows a gene network with three genes, i, j, and k. If there exists protein i produced from gene i, the protein i activates the transcription process from gene k to mRNA k. If there exist both proteins j and k, the transcription process about gene i is repressed. Such activation and repression process are called *protein interactions*.

B. Controlling a gene network

We consider continuous-time switching networks[6], which model gene networks, as an example. The network with n genes is described by Eq. (1), where y_i is a binary variable and indicates if the gene i expresses ($y_i = 1$) or not ($y_i = 0$), where



(i): Interaction relations. (ii): Controlled behavior.

Fig. 3. Simulation results.

x_i is a generalized quantity of the gene i's expression such that the threshold of y_i is set to be 0, and k_i is positive and shows a decomposition rate of gene i. In this model, interaction terms show protein interactions.

Now, we consider 3-dimensional continuous-time switching network with the following interaction terms and decomposition rates:

$$\begin{cases} F_1(\delta) = 1 - 2\delta_{\{2,3\}}, \\ F_2(\delta) = 2\delta_{\{2\}} + 4\delta_{\{3\}} + 4\delta_{\{1,2,3\}}, \\ F_3(\delta) = -3\delta_{\{1\}} + 2\delta_{\{3\}} + 6\delta_{\{1,2\}} - 3\delta_{\{1,3\}}, \end{cases} \tag{32}$$

$$k_1 = 1,\ k_2 = 2,\ k_3 = 3. \tag{33}$$

The network has the following periodic orbit in the sense of modes:

$$\begin{aligned} &(0,0,1) \to (1,0,1) \to (1,0,0) \to \\ &\quad (1,1,0) \to (1,1,1) \to (0,1,1) \to (0,0,1) \to \end{aligned} \tag{34}$$

Now, we control this gene network by adding one gene with $k_4 = 4$ in order to realize the following periodic orbit:

$$\begin{aligned} &(0,0,1) \to (0,0,0) \to (1,0,0) \to \\ &\quad (1,1,0) \to (0,1,0) \to (0,1,1) \to (0,0,1) \to \end{aligned} \tag{35}$$

We consider the following performance function:

$$\begin{aligned} J = &\sum_{j\in N_c} \sum_{j_0 \in 2^{N_c}} \left| w_{j_0}^{(j)} \right| + \sum_{i \in N_o} \sum_{i_0 \in 2^{N_o}} \sum_{j_0 \in 2^{N_c}\setminus\emptyset} \left| w_{i_0 \cup j_0}^{(i)} \right| \\ &+ \sum_{j \in N_c} \sum_{i_0 \in 2^{N_o}\setminus\emptyset} \sum_{j_0 \in 2^{N_c}} \left| w_{i_0 \cup j_0}^{(j)} \right|. \end{aligned} \tag{36}$$

The first, second, and last terms in Eq. (36) are sums of L_1-norm of coefficients in Eqs. (12), (13), and (14), respectively. We use the above performance function (36) which means that the fewer number of interaction is more desirable.

Using the method proposed in Secs. IV and V, we get the following results:

$$\begin{cases} U_1(\delta) = -2\delta_{\{2,4\}} - 2\delta_{\{3,4\}}, \\ U_2(\delta) = -2\delta_{\{4\}} + 2\delta_{\{1,4\}} + 4\delta_{\{2,4\}}, \\ U_3(\delta) = -3\delta_{\{4\}} - 9\delta_{\{1,4\}} + 6\delta_{\{2,4\}} - 3\delta_{\{3,4\}}, \\ F_4(\delta) = 4, \\ Z_4(\delta) = 0. \end{cases} \tag{37}$$

Figure 3 shows the synthesized gene network and its controlled behavior. In Fig. 3(i), dashed lines indicate protein interaction relations in the original gene network, solid ones show the interaction relations which contain the added gene, and "→" and "⊣" mean activation and repression of the indicating gene. Figure 3(ii) shows controlled behavior of the network. The mode transition sequence (35) is achieved.

VII. Concluding Remarks

In this paper, we have proposed a novel control method, which realizes that a PWA system has a given specified symbolic dynamics by adding another PWA system. By using MLD forms, the control problem can be expressed by linear constrained inequalities. We have applied the proposed method to the design of a continuous-time switching network, which models a gene network.

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